



A SIMPLE MODEL INTEGRATING CAPITAL BUDGETING

The authors propose a simple model for integrating credit policy with investment decisions. The model suggests that the length of the credit period should be directly related to the level of capital investment.

WITH TRADE CREDIT

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Corporate financial decisions, whether short- or long-term, result in cash flows spread over time. The conventional approach has been—and in general continues to be—to treat both the timing and magnitudes of cash flows arising from one decision as separable from other decisions, leading to the evaluation of that decision in isolation. Sartoris and Hill¹ referred to this approach as “compartmentalization.” This currently unrelated analysis of corporate decisions may have arisen from separate management by organizationally distinct entities and the categorization of a firm’s assets and liabilities into different packages according to accounting conventions.

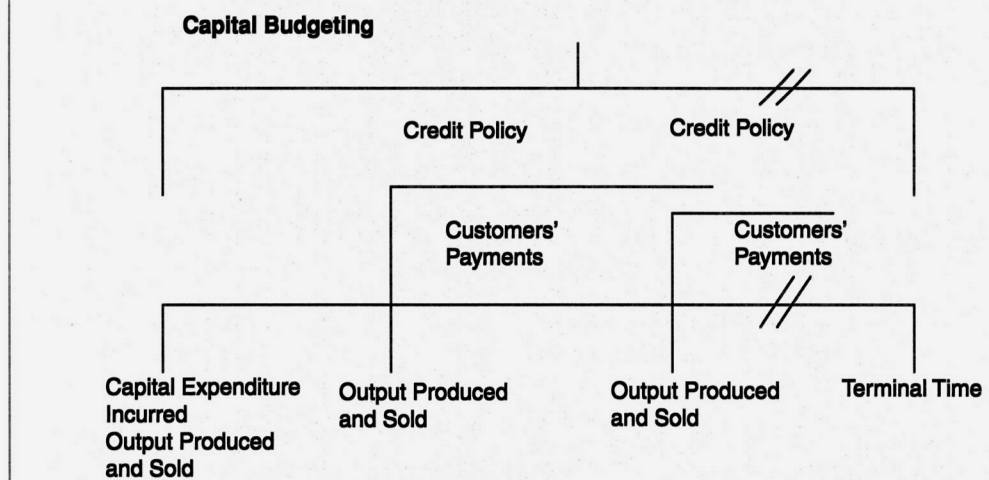
If the timing and/or magnitudes of cash flows of two or more decisions are dependent, financial theory requires that these decisions be considered simultaneously. With respect to investment decisions and credit policy, the cash flow timeline illustrated in Exhibit 1 indicates a joint dependence of cash flows.² Exhibit 1 shows that a capital project generates future cash inflows to the firm whose

timing and magnitudes crucially depend on the firm’s credit policy. Conversely, if the firm estimates to sell significantly more by choosing a suitable credit policy, it should affect the level of capital investment the firm will make. We postulate the investment behavior of a firm by formulating a decision to choose the level of investment expenditure. We assume that the technology is “putty-clay.” This means that the capital is *ex ante* substitutable or variable, but once the decision on the capital investment is made, the level of capital invested cannot be changed. That is, the capital is *ex post* nonsubstitutable. Lim³ shows why this British Cambridge “putty-clay” modeling of capital is most relevant for manufacturing and industrial enterprises in which most capital investment is irreversible short of abandonment.

While the above formulation of a capital budgeting decision is standard, with respect to credit policy, decision variables are numerous as they include the choice of the length of the credit period, a cash discount rate, the length of the discount period, and a series of variables related

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EXHIBIT 1 Cash Flow Timeline for Capital Budgeting and Credit Policy



to collection policy. In this article, we abstract from various dimensions of credit policy to establish a simple framework of integrating capital budgeting with credit policy. The sole credit policy variable considered in this article is the length of the credit period. This formulation of credit policy is not only consistent with integrating research such as that of Kim and Atkins⁴ but is also representative of the typical credit terms offered by small businesses. In this regard, in the next section we shall report some stylized facts about credit policy from the database compiled by the United States National Survey of Small Business Finances (NSSBF) in 1993.⁵

The significance of this research can be viewed in the context of the integrating efforts of several scholars who have attempted to link two or more financial decisions. For example, there have been attempts to integrate corporate finance with macro-economics by Auerbach and King⁶ and Benninga and Talmor.⁷ Froot and Stein⁸ present an integrative framework for the capital allocation and capital structure decisions faced by financial institutions. Schiff and Lieber⁹ have presented a model for the integration of accounts receivable and inventory management. Sartoris and Hill¹⁰ show how cash management, inventory management, and credit management are intricately integrated problems. Kim and Atkins¹¹ evaluate investments in accounts

receivable using a wealth-maximizing framework. Rashid and Mitra¹² and Lim and Rashid¹³ present models of integration of a firm's product pricing decisions with its credit policy. However, among all the integrating attempts, the link between credit policy and investment decision has not been addressed.

The rest of the article is organized as follows. The next section presents a simple model of integration based on some stylized facts about credit policy. A detailed numerical example of the model is provided in the section that follows. Sensitivity analysis with respect to a selected number of exogenous variables is illustrated in the next section. The final section presents the conclusions and implications of the article.

Model

We assume that credit is extended for a period of N days, where N is a variable to be determined. To see the relevance of this simple credit policy, we examine the database compiled by the National Survey of Small Business Finances (NSSBF) in 1993.¹⁴ This dataset focuses on small firms and their major suppliers, in which trade credit is more likely to be a significant form of finance as small firms are more likely to face financial constraints. There are 4,637 small firms in the NSSBF sample. Of these 4,637



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firms, cash discounts (and thus discount periods) were *not offered* to 3,486 firms; in other words, the suppliers to these firms offered them the one-part credit terms that require the determination of the length of the credit period, N , only. The remaining 1,151 firms were offered varying cash discounts for early payment.¹⁵ On the other hand, a credit period was offered to 3,109 small firms. Since major suppliers to small businesses could be firms of any size, this suggests that the credit period was a decision variable for almost three times as many firms as a discount rate or discount period was. The responses on the credit period offered here were categorical, with the response number increasing in the length of the credit period. The longest credit period was more than 90 days (response 9), while the shortest was less than 7 days (response 2). The median and mode credit period was 22 to 30 days (response 5), which was also the mean. In short, although we realize the importance of the two-part credit policy that requires setting up a discount rate and discount period in credit policy, for analytical tractability, we choose to model the one-part credit policy that requires setting up only the credit period.

Next, we assume that investment, made in similar machines, is fully divisible *ex ante*.¹⁶ It is assumed that the initial investment (affecting future replacement/maintenance fixed costs) produces at the end of the j th day an output denoted by Q_j and this output is sold at price per unit P_j , yielding in j th day revenue R_j . Due to perfect competition in the product market, the output price is fixed throughout and without loss of generality, so we let $P = 1$. With P fixed, any price discrimination must come through the credit period, N . Petersen and Rajan¹⁷ have noted that trade credit—in our case, the credit period, N —reduces the effective price to low-quality borrowers. If this is the most price-elastic segment of the market, then changing N is an effective means of price discrimination. A reason why this segment's demand may be more price-elastic is because it is typically credit-rationed. If so, a longer N both lowers the effective price of the good and permits this segment to express its demand. As different firms cater to different customer segments of the same product market, we could

expect N to vary across firms. With these assumptions, the behavioral specification of R_j is the same as that of Q_j . Hence (with I denoting initial capital investment) we postulate that:

$$Q = Q(I, N) \quad (1)$$

That is, the effect of investment I on output Q is positive, but the rate of increase declines as I rises. A similar effect of N on Q is assumed.¹⁸ These postulated effects are intuitively sensible as the effect of I on Q reflects a standard production function relationship while a more liberal credit period (N) is likely to improve sales (and hence production of output), although the increases in sales output will be successively smaller as the credit period is incrementally lengthened.¹⁹

There are two components of costs of production: variable costs, C , and fixed costs, F . Variable costs include labor costs, raw material costs, sales commissions, etc., while fixed costs in our simple model consist of replacement/maintenance costs (or rental costs) and overhead costs (including cost of managing credit department). Obviously, replacement or maintenance costs (or rental costs) increase with the initial capital investment. Since output is a function of initial investment I , there is a positive effect of output on fixed costs. Following these considerations, the variable and fixed costs are specified as follows:

Variable costs, denoted by C :

$$C = C(Q) \quad (2)$$

Fixed costs, denoted by F :

$$F = F(I) \quad (3)$$

The variable costs are postulated to be a convex function of output, which is a standard formulation, while fixed costs are assumed to depend on the level of initial investment.²⁰ It is assumed that both C and F occur at the beginning of each day. It is also assumed that these costs are cash outlays; therefore, the problem of accounts payable management does not exist. In addition, to avoid holding cash for precautionary purposes, we assume perfect certainty about the timings of cash inflows and cash outflows.

It is further assumed that the sales of the firm are credit sales, and a fraction of these sales, denoted by L , is never recovered by the firm. The bad-debt loss ratio is postulated as:

$$L = L(N) \quad (4)$$

The more likely effect of N on L is positive, because the lengthening of credit period may attract those customers who can be classified as “slower” payers, and this group of customers may include a greater proportion of bad debts. However, if the credit quality distribution of customers, attracted by a higher N , is similar to that which pertains to old customers, then changing N will not affect the bad-debt loss ratio.

For discounting future cash flows, we assume the daily discount rate, denoted by k , is a constant. Since the model involves two joint decisions—one of them long-term and the other conventionally treated to be short-term—the issue is what should k be: the short-term rate, or the cost of capital of the project that represents the long-term opportunity cost of capital? For working capital management decisions, the short-term borrowing rate should represent a lower bound for the required discount rate because the firm has the option of (1) repaying short-term debt with any cash inflows generated by the decision; or (2) financing any other short-term financing needs. For capital budgeting, however, the opportunity cost of cash flows is the weighted average cost of capital as these cash flows can be used either to pay down the firm’s long-term liabilities and the shareholders’ equity in the proportion they are raised, or to undertake additional projects of equivalent risk. However, with the joint determination of the two decisions, there is only one series of cash flows, not two. In this situation, credit policy that is tied up with the time horizon of an investment project cannot be treated in a short-term manner. Therefore, the relevant discount rate has to be the weighted average cost of capital. Finally, it is assumed that the firm incurs capital expenditure at time zero. The decision problem can therefore be sketched out on the time line shown in Exhibit 2.

From Exhibit 2, it is evident that with $N > 0$ —that is, when the firm is selling on

a credit basis—the Net Present Value, NPV, is:

$$NPV = \frac{1}{k} \left[\frac{\{(1-L)Q\}}{(1+k)^N} \right] - \frac{1}{k} [(C+F)(1+k)] - I \quad (5)$$

In this equation, the first term is the present value of the after-bad-debt-loss revenue. The second term is the present value of the variable costs and fixed costs, and I stands for initial investment (or start-up costs) at the current date, denoted by zero. The optimal choices of both initial investment I and the length of credit period N (determined initially or at the onset of business) are therefore given by the following objective function:

$$\text{Maximize } NPV = \frac{1}{k} \left[\frac{\{(1-L)Q\}}{(1+k)^N} \right] - (C+F)(1+k) - I \quad (6)$$

The two necessary conditions for a maximum are found by taking the first derivatives of NPV with respect to I and N and setting these expressions to zero.²¹ A joint determination of the two choice variables from the two conditions provides an unconstrained maximum. Therefore, NPV under a joint determination is the highest relative to the NPV under the single choice variable situations of selecting only I or only N . The next section illustrates this simple model with a numerical example.

A numerical example

In this section, a detailed example is worked out to numerically highlight the solution of the proposed simultaneous determination of optimal investment I^* and optimal length of credit period N^* . We recognize that the numerical specifications of the behavioral functions will differ from firm to firm. Therefore, the proposed specifications are simply suggestive.

Revenue, R. With the assumption that $P=1$, we further assume that the revenue of the firm is given by:

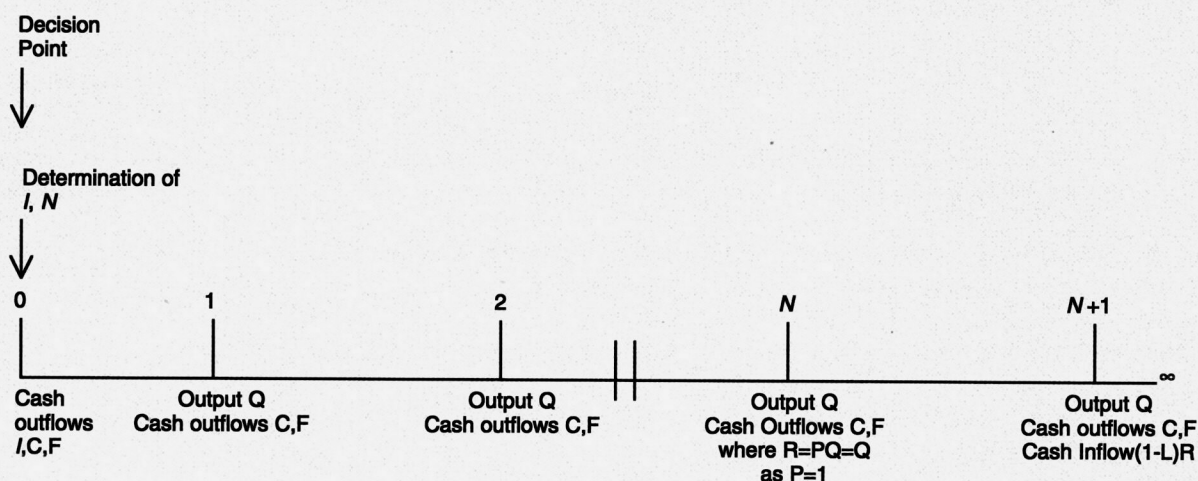
$$Q = A I^{0.5} (1 + 0.03 N) \quad (1)$$

The effect of I on Q is consistent with the Cobb-Douglas production function, while



THE LENGTHENING OF CREDIT PERIOD MAY ATTRACT THOSE CUSTOMERS WHO CAN BE CLASSIFIED AS “SLOWER” PAYERS.

EXHIBIT 2 Decision Point, Output, and Cash Flow Timeline



the size of a positive effect of N on Q depends on the level of production of the firm. This is sensible because the effect of N on Q is expected to be larger for a bigger firm. With equilibrium values (computed below), this specification of Q implies that as N increases by one day, Q increases by 1.5 percent.

Variable Costs of Production, C. For the simplicity of calculation, we assume that C is proportional to Q and the proportionality factor is 0.966. In other words:

$$C = 0.966 Q \quad (2)$$

Fixed Costs of Production, F. F is calculated as follows:

$$F = 100 + 0.0027 I \quad (3)$$

With 365 days/year, the coefficient 0.0027 means fixed costs/year of at least 100 percent of initial investment I since $1/365 = 0.0027$. With equilibrium values (computed below), the autonomous 100 implies that fixed costs per year are an additional 15 percent of I , so total fixed costs are about 115 percent of I each year. As our simple model is perpetual, we have to consider perpetual replacement/maintenance costs. If these are treated as rental costs, then it is certainly reasonable that fixed costs exceed initial invest-

ment (initial rental costs) as fixed costs would also include overhead costs.

Bad-Debt Loss Ratio, L. For simplicity, we assume that L is constant at .01. This is consistent with Scherr,²² who found empirically that L for many firms ranges from 0.008 to 0.014.

Finally, we assume that $A=100$ and the annually compounded cost of capital is 10 percent, which makes the daily cost of fund, k , to be 0.2612 percent, with 365 days/year. We find that the optimal credit period $N^* = 30$ days²³ and the optimal level of investment $I^* = \$264,136$. With $N^* = 30$ days and $I^* = \$264,136$, along with numerical values of the parameters described above in the net present value equation (6), we get:

$$NPV(N^*=30 \text{ days}, I^*=\$264,136) = \$2,612,625$$

Note that the NPV is implausibly larger than initial costs I because this simple model is perpetual. If we exclude the overhead costs and just consider the replacement or maintenance costs (or rental costs) component of fixed costs, which is parameterized to be equal to I each year (365 days), the PV of capital costs over time is $\$(264,136 + 1.0002612^* 264,136 * 0.0027 / 0.0002612) = \$2,995,198$, which is larger than NPV.

EXHIBIT 3 Sensitivity of N^* and I^* to the Variable Cost Per Unit, v^{\dagger}

L = 0.01
 A = 100, and
 F = $100 + 0.0027I$

v	N^*	I^*	NPV
0.96	41.39 days	\$522,821	\$5,546,448
0.966	29.56 days	\$264,136	\$2,612,625
0.97	21.71 days	\$155,593	\$1,381,613

[†] When Annual Discount Rate = 10% (i.e., Daily $k = 0.0002612$).

Sensitivity analysis

In the model proposed above, the exogenous variables are the product price, P ; the variable cost per unit, v (assumed above to be 0.966); the cost of capital, k ; the autonomous scale factor in the firm's revenue function, A ; and the autonomous and coefficient parts of the firm's fixed cost function. The assumed parameters of the model are all the coefficients and exponents in the model. Sensitivity analysis can be conducted with respect to each of these exogenous variables and parameters. While we do not find any added value of the effect of variations in coefficients, exponents and autonomous parts of functions on the optimal N , N^* , and the optimal I , I^* , some additional insight is gained by the sensitivity analysis with respect to v and both v and k together. The results of the calculations are provided in Exhibits 3, 4, and 5.

From Exhibit 3, we find that N^* and I^* (and thus NPV) both decrease as v increases (and vice versa). Since v is the constant marginal variable cost of production, the larger v is, the smaller the profit or contribution margin. In order to remain profitable, a firm has to provide a shorter credit period N to decrease the overall cost. Also, with a smaller profit margin, it is intuitive that the capital investment, I , should decrease.²⁴ Notice that N is particularly sensitive to v . This is because v has a larger impact on the NPV relative to the cost represented by the choice of N . Hence, N has to decrease to a larger extent to compensate for a smaller profit margin when v increases by just a little.

The above point is more clearly demonstrated in Exhibits 4 and 5 where we vary

both v and k , choosing an annual rate of $k=8$ percent in Exhibit 4 and an annual rate of $k=12$ percent in Exhibit 5. As expected, both N and I decrease (increase) in both Exhibits 4 and 5 as v increases (decreases). However, notice that N is more sensitive to v (i.e., N decreases (increases) more in Exhibit 4 than in Exhibit 5 for a similar increase (decrease) in v) when k is lower. This supports our conclusion above that N is particularly sensitive to v because the cost of capital has a smaller impact on NPV than v . When we raise the cost of capital (to 12 percent annually in Exhibit 5), this greater impact of the cost of capital makes N less sensitive to changes in v .

Our numerical results showing an inverse relationship between N and v are consistent with the empirical results of Sartoris and Hill,²⁵ who find that the industry groups with the lowest contribution margins are the most likely to reduce or even eliminate a cash discount. With fixed product price P , the contribution margin $P-v$ would be decreasing in v . Here, we find that the industry groups with the lowest contribution margins (highest v) should decrease the length of the credit period N (to compensate for a higher v). Using the 1993 NSSBF, Lim, Mitra, and Rashid²⁶ found that the magnitude of the cash discount is positively correlated with the length of the credit period, and both are positively correlated with the contribution margin. This suggests that our results on the optimal N are consistent with those of Sartoris and Hill²⁷ and Lim, Mitra, and Rashid²⁸ on the cash discount, where both N and the cash discount are reduced when v increases (or the contribution margin decreases).

EXHIBIT 4 Sensitivity of N^* and I^* to the Variable Cost Per Unit, v , at a Lower Cost of Capital[†]

$$\begin{aligned} L &= 0.01 \\ A &= 100, \text{ and} \\ F &= 100 + 0.0027I \end{aligned}$$

v	N^*	I^*	NPV
0.96	55.38 days	\$699,943	\$9,189,466
0.966	40.77 days	\$342,206	\$4,250,375
0.97	31.01 days	\$195,663	\$2,227,105

[†] When the Annual Discount Rate = 8% (i.e., Daily $k = 0.000210874$).

EXHIBIT 5 Sensitivity of N^* and I^* to the Variable Cost Per Unit, v , at a Higher Cost of Capital[†]

$$\begin{aligned} L &= 0.01 \\ A &= 100, \text{ and} \\ F &= 100 + 0.0027I \end{aligned}$$

v	N^*	I^*	NPV
0.96	32.05 days	\$420,002	\$3,750,765
0.966	22.11 days	\$218,530	\$1,797,033
0.97	15.51 days	\$132,167	\$959,536

[†] When the Annual Discount Rate = 12% (i.e., Daily $k = 0.000310538$).

Finally, and somewhat surprisingly, comparing Exhibits 3 and 4 (and 3 and 5) suggests that I decreases (increases) more in Exhibit 4 than in Exhibit 5 for a similar increase (decrease) in v , when k is lower. Since fixed costs F is increasing in I , this means capital investment (in this perpetual model) is a combination of initial investment (or initial rental cost) I and fixed replacement/maintenance (or rental costs) F . This simple model suggests that the expected contribution margin (affecting the expected operating cash flows) should be an important factor in capital budgeting decisions, perhaps equally important as the cost of capital if the cost of capital is not too high.

Conclusion and implications

A simple model of integration has been proposed. The model has shown that a simultaneous determination of optimal levels of investment expenditure, I , and the length of credit period, N , is optimal. The theoretical derivations of a simultaneous

system in I and N are illustrated by a numerical example. With a fixed product price P in the model, the effective price is lowered only by a liberalization of trade credit terms which, in this model, is represented by lengthening the credit period N . As expected, this model suggests that the length of the credit period should be directly related to the level of capital investment. Next, it suggests that the choice of optimal investment and length of credit period are significantly affected by variation in the variable cost of production, and this sensitivity effect is shown to decline at a higher cost of capital. Finally, the model suggests that the expected contribution margin may be a more important factor in capital budgeting decisions than the cost of capital (when the cost of capital is not too high and the projects are perpetual).

This article has implications for practitioners and researchers alike. It makes a convincing argument to treat investment decisions and credit policy decisions simultaneously. This calls for a change in existing corporate structures that do not permit

such integrated treatment. With increasing attention being paid to greater efficiency and with all the information technology-led structural changes currently occurring in various corporations, it may be the right time for managers to reevaluate whether making investment and credit policy decisions in isolation of each other makes sense. This simple model therefore is timely and provides guidance to managers to determine the optimum levels of investment and lengths of credit period under various situations.

The simple model presented in this article can be extended by incorporating other dimensions of credit policy, such as cash discount rate and cash discount period, to make the model closer to reality. However, given the results of Lim, Mitra, and Rashid,²⁹ which suggest that the cash discount rate and discount period are positively correlated with the credit period (i.e., all credit terms move in tandem), our results here would suggest that the cash discount rate and discount period should be liberalized when capital investment is increased (and vice versa). Other components of capital budgeting—for example, cash flow analysis and managerial options—can also be included in the model. With the growing trend of globalization, organizations throughout the world are setting up their operations in different parts of the world. If the capital project is international, the problem of transfer pricing and tax payments might become very complicated for a firm. Future researchers should incorporate the impact of making investments across different political and economic boundaries into the model. Future research may also explore the possibility of incorporating corporate tax rates and flexible pricing methods into this simple model.

An earlier version of this article was entitled "The Optimal Net Credit Period with Endogenous Putty-Clay Production Technology." Comments from participants at the Northern Finance Association Meeting, Financial Management Association International Meeting, and Asian Finance Conference are appreciated. The authors especially thank Professor William Sartoris, the discussant at the FMA Meet-

ing, for his encouraging remarks and his suggestion to change the title. The authors are also grateful for support from the Social Sciences and Humanities Research Council of Canada. The usual disclaimer applies. ■

NOTES

- ¹ W. L. Sartoris and N. C. Hill, "A Generalized Cash Flow Approach to Short-Term Financial Decisions," *Journal of Finance* (Vol. 38, No. 2, 1983): 349–360.
- ² The standard textbook treatment is to add on to a project some net working capital requirements as a component of non-capital costs in the initial year(s) and reverting back of the accumulated net working capital at the terminal date of the project (see, for example, chapter 10 of A. R. Ross, R. W. Westerfield, B. D. Jordan, and G. R. Roberts, *Fundamentals of Corporate Finance*, 4th Canadian ed. (Toronto: McGraw Hill-Ryerson, 2002), or any other text book on corporate finance). The problem with this approach is that it treats working capital decisions passively. In this paper, the credit period decision, along with the capital expenditure, are jointly determined.
- ³ W. Lim, "Jelly, Putty-Clay and Sunspots," paper presented at the International Association of Financial Engineers Conference, New York, N.Y., 2000.
- ⁴ Y. H. Kim and J. C. Atkins, "Evaluating Investments in Accounts Receivable: A Wealth Maximizing Framework," *Journal of Finance* (Vol. 33, No. 2, May 1978): 403–411.
- ⁵ Federal Reserve Board, *National Survey of Small Business Finances* (NSSBF) (1993). The NSSBF was conducted in 1988, 1993, and 1999 under the guidance of the Board of Governors of the Federal Reserve System and the U.S. Small Business Administration. It targeted nonfinancial, non-farm small businesses that had been in operation for at least a year. Financial data were collected only for the fiscal year in which the survey was conducted. Here, we use data collected in the 1993 survey, which is the latest year for which the data are publicly available.
- ⁶ A. J. Auerbach and M. A. King, "Taxation, Portfolio Choice and the Debt-Equity Ratios," *Quarterly Journal of Economics* (Nov. 1983): 587–609.
- ⁷ S. Benninga and E. Talmor, "The Interaction of Corporate and Government Financing in General Equilibrium," working paper 28-86, Rodney L. White Center for Financial Research, University of Pennsylvania, 1986.
- ⁸ K. A. Froot and J. C. Stein, "Risk Management, Capital Budgeting, and Capital Structure Policy for Financial Institutions: An Integrated Approach," *Journal of Financial Economics* (Jan. 1998): 55–82.
- ⁹ M. Schiff and Z. Lieber, "A Model for the Integration of Credit and Inventory Management," *Journal of Finance* (Vol. 29, No. 1, 1974): 133–141.
- ¹⁰ Sartoris and Hill, *op. cit.* note 1.
- ¹¹ Kim and Atkins, *op. cit.* note 4.
- ¹² M. Rashid and D. Mitra, "Price Elasticity of Demand and an Optimal Cash Discount Rate in Credit Policy," *Financial Review* (Vol. 34, No. 1, 1999): 113–126.
- ¹³ W. Lim and M. Rashid, "An Operational Theory Integrating Cash Discount and Product Pricing Policies," *Journal of American Academy of Business* (Vol. 1, No. 2, March 2001): 282–289.
- ¹⁴ Federal Reserve Board, *op. cit.* note 5.
- ¹⁵ Credit terms with a cash discount for early repayments are known as the two-part credit policy. This policy requires the determination of three credit policy variables: the cash discount rate, the length of the discount period and the length of the credit period.



THE SIMPLE MODEL PRESENTED IN THIS ARTICLE CAN BE EXTENDED BY INCORPORATING OTHER DIMENSIONS OF CREDIT POLICY, SUCH AS CASH DISCOUNT RATE AND CASH DISCOUNT PERIOD, TO MAKE THE MODEL CLOSER TO REALITY.

¹⁶We recognize that in the so-called "real world," most investments are not infinitely divisible. For this paper this is a moot point because by suitably choosing units of measuring investment, this investment opportunity schedule can be made more continuous and smooth.

¹⁷M. Petersen and R. G. Rajan, "Trade Credit: Theories and Evidence," *Review of Financial Studies* (Vol. 10, 1997): 661-691.

¹⁸Mathematically, we assume that $\delta Q/\delta I > 0$, $\delta Q/\delta N > 0$, $\delta^2 Q/\delta I^2 \leq 0$, and $\delta^2 Q/\delta N^2 \leq 0$.

¹⁹The lengthening of the credit period is equivalent to a decrease in the effective price to customers. Therefore, the size of the effect on sales depends on the price elasticity of demand of the firm's product or service. Only when the firm's product or service has perfectly inelastic demand will there be no effect on demand; otherwise demand will go up.

²⁰Mathematically, we assume that $\delta C/\delta Q > 0$ and $\delta^2 C/\delta Q^2 \geq 0$ and $\delta F/\delta I > 0$.

²¹A sufficient condition for a relative maximum requires the satisfaction of the second order derivatives of the objective function in equation (6). But, given our assumptions of the concavity of the production function in equation (1) and convexity of cost functions in equations (2) and (3), the second order conditions are automatically satisfied.

²²F. C. Scherr, *Modern Working Capital Management: Text and Cases* (Englewood Cliffs, N.J.: Prentice Hall, 1989).

²³Rounding to the nearest integer.

²⁴This comes about from our assumptions about the revenue (production) and cost functions, and the neoclassical economics optimizing condition that marginal revenue equals marginal cost.

²⁵W. L. Sartoris and N. C. Hill, "The Relationship Between Credit Policies and Firm Financial Characteristics," in Y. H. Kim and V. Srinivasan (eds.), *Advances in Working Capital Management*, Vol. 1 (Greenwich, Conn.: JAI Press, 1988).

²⁶W. Lim, D. Mitra, and M. Rashid, "Determinants of the Cash Discount Rate in Credit Policy," in R. K. Gupta and M. D. Skipton (eds.), *Proceedings of the ASB 2000 Conference* (St. John's, Newfoundland: Actes, Memorial University of Newfoundland, 2000): 91-100.

²⁷Sartoris and Hill, *op. cit.* note 25.

²⁸Lim, Mitra, and Rashid, *op. cit.* note 26.

²⁹*Ibid.*

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